

HIGH SCHOOL MATHEMATICS CONTESTS

Math League Press, P.O. Box 17, Tenafly, New Jersey 07670-0017

Contest Number 1

November 1, 1988

Name _____ Grade Level _____ Score _____

Time Limit: 30 minutes

Answer Column

1-1. In a right triangle, the lengths of the legs are 333 and 444. What is the length of the hypotenuse?

1-1.

1-2. Ann and Bob take turns in a game. At a turn, a player takes one or two coins from a cup that at first holds six coins. The player taking the last coin wins. If Ann goes first and both play perfectly, who will win?

1-2.

1-3. On a clock with hour, minute, and second hands, the second hand is 14 cm long. In 71 minutes, the tip of the second hand travels a distance of $k\pi$ cm. What is the value of k ?

1-3.

1-4. Express, as a fraction in lowest terms, the value of the following product of 99 factors:

1-4.

$$\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right) \times \dots \times \left(1 - \frac{1}{n+1}\right) \times \dots \times \left(1 - \frac{1}{100}\right).$$

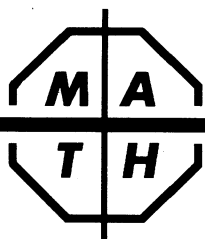
1-5. At noon, a train leaves New York for Toronto while another leaves Toronto for New York. It takes one train 8 hours and the other 22 hours to make the trip. Both maintain constant speeds. At what time do they pass?

1-5.

1-6. What are all ordered triples of real numbers (x,y,z) which satisfy

1-6.

$$\begin{aligned}(x + y)(x + y + z) &= 120, \\(y + z)(x + y + z) &= 96, \\(x + z)(x + y + z) &= 72?\end{aligned}$$



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Contest Number 3

January 10, 1989

Name _____ Grade Level _____ Score _____

Time Limit: 30 minutes

Answer Column

3-1. What is the units' digit of

$$1 + 9 + 9^2 + 9^3 + \dots + 9^{n-1} + \dots + 9^{1988} + 9^{1989}?$$

3-1.

3-2. Winkin said "We're all liars," Blinkin replied, "Only you are a liar," and Finkin said "You're both liars." If each of them always tells the truth or always lies, how many of the three are liars?

3-2.

3-3. What are all real values of x which satisfy

$$x + |x| = 0?$$

3-3.

3-4. Two linear functions are said to be inverses if their graphs are reflections of each other across the line $y = x$. What are the three different ordered pairs of real numbers (a,b) for which $y = ax + b$ is its own inverse?

3-4.

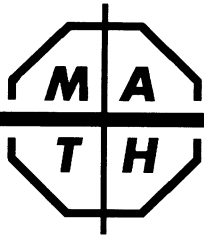
3-5. What is the (simplified) value of k for which the larger root of $x^2 + 4x + k = 0$ is

$$\sqrt{2+\sqrt{3}}\sqrt{2+\sqrt{2+\sqrt{3}}}\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{3}}}}\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{3}}}}?$$

3-5.

3-6. One circle has a radius of 5 and its center at $(0,5)$. A second circle has a radius of 12 and its center at $(12,0)$. What is the length of a radius of a third circle which passes through the center of the second circle and both points of intersection of the first 2 circles?

3-6.



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Contest Number 5

March 7, 1989

Name _____ Grade Level _____ Score _____

Time Limit: 30 minutes

Answer Column

5-1. Single copies of a book cost \$16 each, but purchasers of 20 or more books pay only \$13 per book. What are all values of $n < 20$ for which one could buy 20 books at a lower total cost than one could buy exactly n books?

5-1.

5-2. What are both values of x which satisfy

$$1989^{10} \times 1989^{20} \times 1989^{30} \times 1989^{40} = (1989^x)^x?$$

5-2.

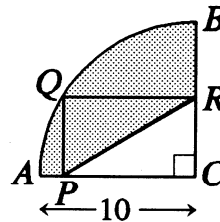
5-3. I pay for a \$1 burger with 48 coins and get no change. What is the largest number of nickels I could use?

5-3.

5-4. What is the value of $b > 0$ for which the region bounded by both the x -axis and $y = -|2x| + b$ has an area of 72?

5-4.

5-5. The quarter-circle shown at the right has center C and radius 10. If the perimeter of rectangle $CPQR$ is 26, what is the perimeter of the shaded region?



5-5.

5-6. What are all positive numbers x which satisfy the equation

$$\log_2 x \log_4 x \log_6 x = \log_2 x \log_4 x + \log_2 x \log_6 x + \log_4 x \log_6 x?$$

5-6.