Math League News

■ Our Calculator Rule Our contests allow both the TI-89 and HP-48. You may use any calculator without a QWERTY keyboard.

■ Use the Internet to View Scores or Send Comments to comments@mathleague.com. You can see your results at *www.mathleague.com*!

■ <u>PLEASE NOTE!</u> Error on Printed Solution to Question 6 on Contest #6 Due to an error in our formatting process prior to printing, the final character of the official answer to Question 6 on Contest 6 was omitted. The correct answer should be the integer currently boxed on the solution followed by "π," and not simply the integer by itself. Please give credit to students who include π in their answer to this question, and NOT to any answer that does not include π (or the correct numerical value of the integer currently boxed as the solution multiplied by the value of π). We are sending an email to everyone on our mailing list in order to point out the issue, and will note the discrepancy on our website.

■ Dates of Final HS Contest and Algebra Contest Our final contest of this school year is March 12 (with an alternate date of March 19). In addition, this year happens to be the 19th year of our annual April *Algebra Course I* contest. There's still time for your school to register! Go to **www.mathleague.com**.

■ 2013-2014 Contest Dates We schedule the six contests to be held four weeks apart (mostly) and to end in March. Next year's contest (and alternate) dates, all Tuesdays, are October 15 (Oct. 22), November 12 (Nov. 19), December 3 (Dec. 10), January 14 (Jan. 21), February 11 (Feb. 18), and March 11 (Mar. 18). Have a testing or other conflict? Now is a good time to put an alternate date on calendar!

■ Rescheduling a Contest and Submitting Results Do you have a scheduling problem? If school closings or testing days mandate contest rescheduling, our rules permit you to use an alternate contest date. Try to give the contest the week after the regularly scheduled date. If scores are late, attach a brief explanation. Late scores unaccompanied by such an explanation will not be accepted.

■ End-of-Year Awards Engraving of awards begins March 31st. We give plaques to the highest-scoring school in each region and to the 2 schools and the 2 students with the highest totals in the entire League. *Winning schools must submit their results to our Internet Score Report Center by Match 31st.* Results submitted later *cannot* be used to determine winners. A teacher once asked, "*Has there been any thought to using enrollment figures to divide the schools into two divisions? Personally, I don't care whether we ever receive any team recognition, as my students enjoy the mathematical challenges provided.*" Our groupings are not organized to "even out" the competition. Competition is one feature of our academic enrichment activity, but enrichment should be the main goal. Only a few schools can expect to win, but all schools can profit.

■ General Comments About Contest #5: Denes Jakob said, "As always we enjoyed the challenge of your contest." Robert Morewood said, "Thank you for another amusing contest!" Mark Luce said, "Tough contest with some challenging problems, especially those last two!" Chip Rollinson said, "Tough round. I don't remember when the last time NONE of my students did not get 5 or above. Several were VERY close on the ones they missed. Careless errors were many. Great set of problems nonetheless." Dick Gibbs said, "All in all, a very nice contest!"

■ Question 5-1: Comment and Alternate Solution Robert Mark Luce said, "My best student, who has often had perfect scores on these contests, only got a 5 this time, because he misinterpreted the first problem--the easiest one. He read the 'hexagon's rightmost vertex' as meaning the 'rightmost vertex on the *x*-axis.' And got the answer of 16, which would be correct, with that interpretation. But as I told him, if that was what you wanted, you would have said that." Several of you wrote in with an alternate solution in which you recognized that the hexagon can be divided into 6 equilateral triangles. The diagonal running across the middle of the hexagon has length 4, so each triangle is of side length 2, and the perimeter of the hexagon is 12. Versions of this alternate solution were sent in by Denes Jakob, Dick Gibbs, and Lew Davidson.

■ Question 5-4: Comments and Alternate Solution Robert Morewood said, "I found #4 particularly nice." Chip Rollinson said, "I did not see the answer to #5-4 coming!" Mark Luce said, "[A] student had the right answer (1/2013) for Problem 4, but then changed it, for some odd reason, to -2013. I have no idea why." Dick Gibbs offered this clever (partially) alternative solution: "Once we have $S = (\cos^2(x))/(1 - \cos^2(x))$, the astute solver is sure to note that $\cos^2(x) = 1 - \sin^2(x)$ and that $1 - \cos^2(x) = \sin^2(x)$, so that $S = (1 - \sin^2(x))/\sin^2(x)$, which is just the reciprocal of the sine series."

■ Question 5-5: Alternate Solution Chip Rollinson proposed an alternative approach, saying "Here's how I did it.
$C(n,1)\times C(100-n,1) = C(n,2) + C(100-n,2) \Leftrightarrow$
$n\times(100-n) = n\times(n-1)/2 + (100-n)(99-n)/2 \Leftrightarrow$
$100n - n^2 = (n^2 - n)/2 + (9900 - 199n + n^2)/2 \Leftrightarrow$
$200n - 2n^2 = n^2 - n + 9900 - 199n + n^2 \Leftrightarrow$
$0 = 4n^2 - 400n + 9900 \Leftrightarrow$
$0 = n^2 - 100n - 2475$ (same equation that you arrived at)."

■ Question 5-6: Comments and Alternate Solutions Several of you suggested that the question might be answered more easily by adding 1 to each side and factoring, such that $N + 1 = (a - 1)(b - 1)$. This approach makes it clear very quickly that $N + 1$ must be prime to restrict the possibilities to a unique solution. Robert Morewood said, "From the familiar Postage Stamp Problem, I knew immediately that $(N + 1) = (a - 1)(b - 1)$, but how were the students to discover that? The Long Division which they have just done!" Steve Gregg said, "Perhaps a better solution to 5-6 is to use what the Art of Problem Solving folks call 'Simon's Favorite Factoring Trick.' Add 1 to both sides of the original equation ... From there the argument proceeds as in the published solution, but I think using this factoring 'trick' is a much easier way to get there." Dick Gibbs and Chip Rollinson also mentioned this approach. Chip Rollinson mentioned some other alternatives as well, saying "I had several students brute force this problem (with some good logic to eliminate possibilities) to arrive at the correct answer. One of my students noticed that if he took the equation and added 1 to each side ... I arrived at the same conclusion slightly differently. I noticed that $ab - (a + b)$ is the sum of the linear and constant coefficients of a quadratic function with zeros a and b ... $f(x) = x^2 - (a + b)x + ab$. Therefore $f(1) = 1 - (a + b) + ab = N + 1$. In order for the quadratic to factor uniquely (with whole number roots), $N + 1$ must be prime."

Statistics / Contest #5

Prob #, % Correct (*all reported scores*)

5-1	67%	5-4	17%
5-2	89%	5-5	12%
5-3	79%	5-6	18%