Math League News

■ Our Calculator Rule Our contests allow both the TI-89 and HP-48. You may use any calculator without a QWERTY keyboard.

■ Use the Internet to View Scores or Send Comments to comments@mathleague.com. You can see your results at *www.mathleague.com*!

■ Dates of Final HS Contest and Algebra Contest Our final contest of this school year is March 17 (with an alternate date of March 24). In addition, this year happens to be the 20th year of our annual April *Algebra Course I* contest. There's still time for your school to register! Go to **www.mathleague.com**.

■ Carefully Check Your Contest Package—Disregard Incorrect "2013-2014" Designation Without opening it, please check that the remaining envelope is number 6. If you're missing the envelope, e-mail *dan@mathleague.com* with your name, the school's name, and the full school address, and we'll mail you another set of contests right away. Please note that the envelope containing the contests has the year's schedule printed on them. <u>While the schedule is correct, the heading has the wrong year.</u> Please disregard the "2013-2014" heading.

■ 2015-2016 Contest Dates We schedule the six contests to be held four weeks apart (mostly) and to end in March. Next year's contest (and alternate) dates, all Tuesdays, are October 13 (Oct. 20), November 10 (Nov. 17), December 8 (Dec. 15), January 12 (Jan. 19), February 9 (Feb. 16), and March 15 (Mar. 22). Have a testing or other conflict? Now is a good time to put an alternate date on calendar!

■ Rescheduling a Contest and Submitting Results Do you have a scheduling problem? If school closings **or testing days** mandate contest rescheduling, our rules permit you to use an alternate contest date. Try to give the contest the week after the regularly scheduled date. If scores are late, attach a brief explanation. Late scores unaccompanied by such an explanation will not be accepted.

■ End-of-Year Awards Engraving of awards begins March 31st. We give plaques to the highest-scoring school in each region and to the 2 schools and the 2 students with the highest totals in the entire League. *Winning schools must submit their results to our Internet Score Report Center by March 31st.* Results submitted later *cannot* be used to determine winners. A teacher once asked, "*Has there been any thought to using enrollment figures to divide the schools into two divisions? Personally, I don't care whether we ever receive any team recognition, as my students enjoy the mathematical challenges provided.*" Our groupings are not organized to "even out" the competition. Competition is one feature of our academic enrichment activity, but enrichment should be the main goal. Only a few schools can expect to win, but all schools can profit.

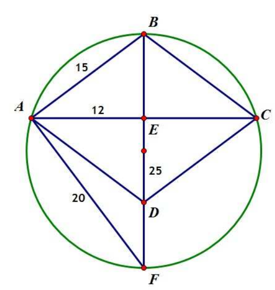
■ General Comments About Contest #5: Chip Rollinson said, "Thanks for the contest ... Another fun set of questions." Abdulkerim Akyalcin said, "Thank you so much for another great set of problems. By the way, [for the] first time, we had a student receive a perfect score. We are improving. :) Thanks." Mark Luce said, "I loved all the ... problems, especially the two geometry problems." Dean Ballard said, "Another good contest!"

■ Question 5-2: Comment Joseph Bak said, "[Question 5-2] could have been solved if only the first condition: slope=y intercept were given ... with the added note that they are nonzero."

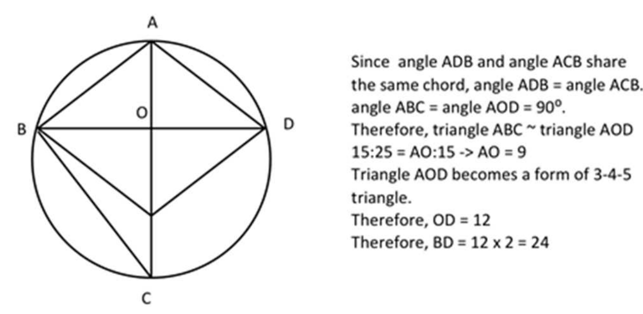
■ Question 5-3: Comments Mark Luce said, "Several students solved Problem 3 with trigonometry ... instead of the (to my mind) simpler algebraic method used on the solution sheet." Ben Jordan said, "I did not accept a decimal answer for question 3, although perhaps it could have been worded that the answer had to be exact." Decimal answers correct to 4 or more significant digits that are correctly rounded should be marked correct.

■Question 5-4: Comment Chip Rollinson said, "For #5-4, I had several kids guess and check... and (2,3) was their first guess!"

■ Question 5-5: Comments and Alternative Solutions Mark Luce said, "One student solved Problem 5 using the Law of Cosines, rather than the method given." Chip Rollinson also mentioned that several students used the Law of Cosines, and said, "I had a couple of students make the assumption that there would be a 9-12-15 triangle. Perhaps slightly messier numbers would have been better since calculators are allowed. ... I found it interesting that given any 3/4 inscribed rhombus with rational length sides s in a circle with a rational length diameter d, the two parts of the diameter containing one diagonal and cut by the other diagonal will always have rational lengths s^2/d and d-s^2/d." Benjamin Jordan said, "Question 5 was solved by trigonometry. Label the vertices of the rhombus clockwise from the top *ABCD*. Label the center of the circle O. Label the center of the rhombus F. From the perimeter, $AB = BC = CD = DA = 15$; $OA = OD = 12.5$, therefore triangle *AOD* is isosceles. Drop the bisector of $\angle AOD$ to the midpoint of *DA*, point *E*. Triangle *DOE* is right. Then $DE = 7.5$ and angle *DOE* $= \sin^{-1}(7.5/12.5) = 36.87$ $\angle AOD = 2^*\angle DOE = 73.74$ Now consider right triangle *DFO*. $\angle FOD = \angle AOD = 73.74$ and $OD = 12.5$, so $DF = 12.5 \sin 73.74 = 12$. Length of $BD = 2(DF) = 24$." Paul Johnson said, "Draw a chord from right-hand vertex of rhombus to bottom of diameter. This forms a right triangle, such that this new side has length 20. (Other Leg=15, Hypotenuse=25.) The upper triangle of the rhombus is similar to this new right triangle (altitude to hypotenuse), so to find half of the requested diagonal, solve proportion: $a/15 = 20/25$. (Proposed by Sam Robison.)" Mark Fowler said, "Place the center of the circle at the origin (0, 0). The upper vertex of the rhombus will be at the point (0, 12.5). Using this as the center of a second circle with radius 15 (the length of the rhombus side), we can find the intersection (x, y) of the two circles through substitution. Since the rhombus is symmetric about the y-axis, the longer diagonal measure is $2y = 24$." Dean Ballard, referring to the diagram below, said, "One of my students (Grant H.) found a nice solution to problem 5-5. He added segment *AF*, noting that angle *BAF* intercepts a diameter, and is therefore a right angle. So, by the Pythagorean Theorem, $AF = \text{sqrt}(25^2 - 15^2) = 20$. (similar to 3-4-5) *AE* is an altitude of a right triangle, so triangle ABE is similar to triangle *FBA*. That makes $AE = AF \times (15/25) = 12$, so $AC = 24$."



Chip Rollinson's student Victor (Jung Soo) Chu had a similar alternative solution as below:



■ Question 5-6: Comments and Alternative Solution Chip Rollinson said, "For #5-6, I loved the problem. It was a good one for students to puzzle through. I'm not sure if the Titanic needed to be referenced since 1500 people died on it. Try to imagine the 2115 Math League Contest with a World Trade Center cartoon." Mark Luce said, "I think it would take me the better part of a weekend to figure out Problem 6. That problem struck me as a real gut-wrencher. I did have one student who solved it and I am not sure how. I am going to be asking him!" Ben Jordan said, "Question 6 can also be solved by breaking the rooms into three groups as you suggest and then recognizing that each group has a distinct set of keys with no cross over. Hence the number of ways of organizing group 1 is 4!, the number of ways of organizing group 2 is 5! and group 3 is 5!. Using the counting principle, the number of ways we can arrange all the rooms/keys is 5!5!4! = 345600."

Statistics / Contest #5

Prob #, % Correct (*all reported scores*)

5-1	92%	5-4	44%
5-2	76%	5-5	30%
5-3	59%	5-6	20%