Math League News

■ Our Calculator Rule Our contests allow both the TI-89 and HP-48. You may use any calculator without a QWERTY keyboard.

■ Use the Internet to View Scores or Send Comments to comments@mathleague.com. You can see your results at *www.mathleague.com*!

■ Dates of Final HS Contest and Algebra Contest Our final contest of this school year is March 20 (with an alternate date of March 27). In addition, this year happens to be the 23rd year of our annual April *Algebra Course I* contest. There's still time for your school to register! Go to **www.mathleague.com**.

■ 2018-2019 Contest Dates We schedule the six contests to be held four weeks apart (mostly) and to end in March. Next year's contest (and alternate) dates, all Tuesdays, are October 16 (Oct. 23), November 13 (Nov. 20), December 11 (Dec. 18), January 10 (Jan. 17), February 12 (Feb. 19), and March 19 (Mar. 26). Have a testing or other conflict? Now is a good time to put an alternate date on calendar!

■ Rescheduling a Contest and Submitting Results Do you have a scheduling problem? If school closings **or testing days** mandate contest rescheduling, our rules permit you to use an alternate contest date. Try to give the contest the week after the regularly scheduled date. If scores are late, attach a brief explanation. Late scores unaccompanied by such an explanation will not be accepted.

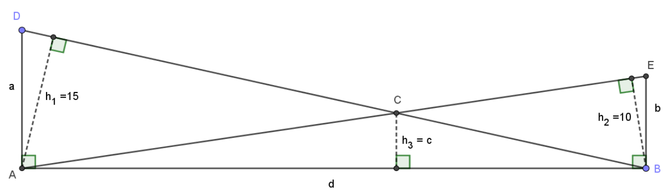
■ End-of-Year Awards Engraving of awards begins March 27th. We give plaques to the highest-scoring school in each region and to the 2 schools and the 2 students with the highest totals in the entire League. *Winning schools must submit their results to our Internet Score Report Center by Match 31st.* Results submitted later *cannot* be used to determine winners. A teacher once asked, "*Has there been any thought to using enrollment figures to divide the schools into two divisions? Personally, I don't care whether we ever receive any team recognition, as my students enjoy the mathematical challenges provided.*" Our groupings are not organized to "even out" the competition. Competition is one feature of our academic enrichment activity, but enrichment should be the main goal. Only a few schools can expect to win, but all schools can profit.

■ General Comments About Contest #5: James Conlee said, "Contest #5 was appropriately difficult and offered something for all students. Love it." Vivian Nelson said, "Thanks for a few questions that many students could answer. The last two contests were extremely frustrating for my students." Chip Rollinson said, "A relatively easy set of questions up until #5 and #6 which were appropriately more difficult."

■ Question 5-3: Comments and Appeals (accepted) Joseph Li said, "Problem 3 is interesting. I saw one student drew the third beaver on the test paper. It is more difficult than problem 4 and 5 for my students." Jon Mormino pointed out two possible interpretations of the question's wording when he said, "On problem #3, 'twice as likely to make the tree fall' is ambiguous. 1) 'Twice as likely to make the tree fall [on any given strike].' or 2) 'Twice as likely to make the tree fall [when it eventually falls].'" Several other advisors, including Benjamin Dillon, Dan LaVallee, and Paula Woodward appealed for the interpretation of the question that requires the use of an infinite series, reading the question as though it had said " . . . on EACH TURN the second beaver is twice as likely as the first to make the tree fall . . .." The answer under this interpretation is 120/253, which will be given credit. The logic leading to that answer was submitted by student Ayush Kamat, who said, "Clearly, at any given turn, the probability that the third beaver makes the tree fall is 4/7, but the problem states that they take turns biting the tree. Hence the probability that the third beaver bites it down on his first pass (which means that neither the first nor the second bit it down) is $P(1) = 6/7\times5/7\times4/7 = 120/343$, the probability that he bites it down on his second pass is $P(2) = 6/7\times5/7\times3/7\times P(1)$. We can easily see that the probability that the third beaver bites down the tree on his nth pass is described by the recursion $P(n) = 6/7\times 5/7\times3/7\times P(n-1) = 6/7\times5/7\times(3/7)^{(n-1)}\times P(1)$, hence we can write $P(\text{third beaver}) = P(1) + P(2) + P(3) + \ldots = P(1)/(1-(6/7\times5/7\times3/7))$ (geometric sum formula) $= 120/(343 - 6\times5\times3) = 120/253$."

■ Question 5-4: Appeal (Accepted) Heidi Meakin appealed on behalf of a student who answered $((4-(m-4)),n)$. Since this answer is mathematically equivalent to the official answer, the student should be given credit.

■ Question 5-5: Comments and Alternative Solution Wes Loewer said "Say the triangle in question is ΔABC shown below with the three altitudes labeled $h_1=15$, $h_2=10$, and $h_3=c$. As side AB gets larger, angle C gets larger, and altitude h_3 gets smaller. This situation reminded me of a somewhat similar looking arrangement of two right triangles that share a leg, ΔDAB and ΔEBA. It can be shown that $1/a + 1/b = 1/c$, or $c = ab/(a+b)$. In the picture shown, $a>15$ and $b>10$, but in the case where $a = 15$ and $b = 10$, then $c = 6$. As side AB gets larger and larger, a and b approach 15 and 10 respectively, so c will always be larger than 6 but will approach 6 (ie,



as $d\to\infty$, $c\to6+$). So c can be arbitrarily close to 6, but will never reach 6, so the smallest integer length must be 7."

■ Question 5-6: Comments and Alternative Solution Chip Rollinson said, "I have one problem with #6...if a student had a CAS calculator, they could quickly find the irrational roots of the quartic equation which makes the problem much easier than intended since factoring the quartic was quite non-trivial. At some schools, all students have CAS calculators. At our school only a handful of our students have a CAS calculator. As a result, I feel that question #6 should not be counted since it gives some schools/ students an unfair advantage." The problem was reviewed by math professors at Stanford University. They ruled as appeals judges that the problem was valid and should not be removed. Joseph Li said, "Problem 6 is a very nice problem that combines polynomials and trigonometry. It is very challenging but every step is natural." Wes Loewer proposed an alternative way to solve 5-6, saying "Once you get $\cos(2\theta) = -13+8\text{sqrt}(3)$, rearranging to get $(\cos(2\theta)+13)^2 = (8\text{sqrt}(3))^2$ does not seem intuitive to me at all. A more intuitive approach is to say that if $-13+8\text{sqrt}(3)$ is a root, then the conjugate $-13-8\text{sqrt}(3)$ must also be a root since a and b are integers. Just expand $(x-(-13+8\text{sqrt}(3)))\times(x-(-13-8\text{sqrt}(3)))$ to get $x^2 + 26x - 23$."

Statistics / Contest #5

Prob #, % Correct (*all reported scores*)

5-1	85%	5-4	48%
5-2	69%	5-5	10%
5-3	59%	5-6	5%