2012-2013 ALGEBRA COURSE 1 CONTEST SOLUTIONS

Answers

23. If $(x-2)^2 = 1600$, $x-2 = \pm 40$. Thus $x = 42$ or -38, and $x-4 = 38$ or -42.

A) -42 B) -34 C) 34 D) 36

23. A

24. Since the prime factorization of 260 is (2)(2)(5)(13), the least possible value of x is 13.

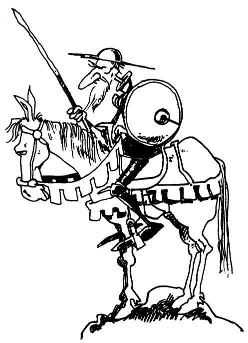
A) 10 B) 13 C) 26 D) 30

24. B

25. Avg. speed = (total dist./total time), so Don Q's avg. speed is $(60+60)/[60/(3r)+60/(6r)] = 120/(30/r) = 4r$.

A) $4r$ B) $4.5r$ C) $5r$ D) $5.5r$

25. A



26. If the integer is $10t+u$, then the difference be-between this integer and the integer with the digits reversed is $(10t+u)-(10u+t) = 9t-9u = 36$. Dividing by 9, $t-u = 4$.

A) 4 B) 6 C) 8 D) 9

26. A

27. My sister has s dollars, and I have d dollars more than she has. If together we have a total of t dollars, then $s+(s+d) = t$, so $2s = t-d$ and $s = (t-d)/2$.

A) $t-2d$ B) $\frac{t}{2}-d$ C) $t-\frac{d}{2}$ D) $\frac{t-d}{2}$

27. D

28. Choice D is the product of 3 consecutive integers, so it's divisible by 3.

A) $x(x-3)(x-6)$ B) $x(x+3)(x-3)$ C) $x(x+7)(x-2)$ D) $x(x+1)(x-1)$

28. D

29. The expression $\frac{2x+1}{3x-3}$ becomes $\frac{2(\frac{4}{x})+1}{3(\frac{4}{x})-3} = \frac{\frac{8}{x}+1}{\frac{12}{x}-3} = \frac{8+x}{12-3x}$.

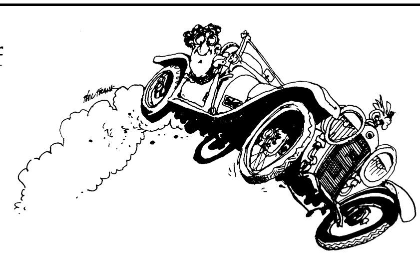
A) $\frac{2x+1}{3x-3}$ B) $\frac{3x-3}{2x+1}$ C) $\frac{8+x}{12-3x}$ D) $\frac{12x-3}{8x+1}$

29. C

30. The inequality is true if $x = -3$ or -4. If $x < -4$ or $-3 < x < 5$, it is false. If $x = 6$ or 7, it is true.

$$\frac{(x+3)(x+4)}{x-5} \geq 0.$$

My car has 4 passengers.



A) 2 B) 3 C) 4 D) 5

30. C

The end of the contest **A**

Visit our Web site at http://www.mathleague.com

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Information & Solutions

Spring, 2013

A

Directions for Grading

- **Date** You may give this contest any time after April 15. The *Algebra Course 1 Contest* is for use in your own school or district. We've enclosed a registration form for next year. Instructions for optionally submitting results are included on a separate sheet entitled "*Using the Score Report Center*."
- **Urgent questions?** Write to comments@mathleague.com, or call 1-201-568-6328 or 1-516-365-5656.
- **Scores** Remind students that *this is a contest, and not a test*—there is no "passing" or "failing" score. Few students score as high as 24 points (80% correct); students with half that, 12 points, *should be commended!*
- **Solutions** Detailed solutions appear in each question box, and letter answers are in the *Answers* columns on the right. You may copy this solution key and give a copy to every student who took this contest.
- **Awards** The original contest package contained 1 book award (and a bookplate you should affix to the book's inside front cover) for the 1st place student. We also enclosed 5 *Certificates of Merit*—1 each for the runner-up on each grade level, plus extras for ties.
- **Additional Book Awards & Additional Certificates** If you want to give more than 1 book award, you may purchase additional books as described below. Do you need more Certificates of Merit? If so, send your name, school, and school mailing address to our mailer at: ***Math Certificates, P.O. Box 17, Tenafly, NJ 07670-0017.*** *Include a self-addressed, stamped envelope (**2 stamps required**) large enough to hold certificates.*

The school's top scorer will receive the book *Math Contests—High School (Vol. 4)*. Other high scorers will receive Certificates of Merit. In any one school year, no student may win both a book and a certificate. The book and certificates were in the original contest package.

If needed, duplicate book awards may be ordered as described below.

Eighteen books of past contests, *Grades 4, 5, & 6* (*Vols. 1, 2, 3, 4, 5, 6*), *Grades 7 & 8* (*Vols. 1, 2, 3, 4, 5, 6*), and *High School* (*Vols. 1, 2, 3, 4, 5, 6*), are available, for \$12.95 per volume, from Math League Press, P.O. Box 17, Tenafly, NJ 07670-0017.

1. If $x = 2013$, then $(x - 2012)^{(x - 2013)} = (2013 - 2012)^{(2013 - 2013)} = 1^0 = 1$.

   A) 0 B) 1 C) 2 D) 10

   1. B

2. If $a = 5$, then $4a^3 - 3a^2 + 2a - 1 = 4(5)^3 - 3(5)^2 + 2(5) - 1 = 500 - 75 + 10 - 1$.

   A) 39 B) 125 C) 434 D) 586

   2. C

3. Fred and Ginger danced for $\dfrac{2013}{x}$ hours last year. Since 2013 is not divisible by 13, x *cannot* be 13.

   A) 3 B) 11 C) 13 D) 61

   3. C

4. We may rewrite $x^2 - 4x - 12$ as $(x - 6)(x + 2)$, so $x + 2$ is a factor.

   A) $x + 2$ B) $x - 2$ C) x D) $x - 8$

   4. A

5. $2^{400} + 2^{400} = 2(2^{400}) = (2^1)(2^{400}) = 2^{400+1} = 2^{401}$.

   A) 2^{401} B) 2^{800} C) 4^{400} D) 4^{800}

   5. A

6. If $\dfrac{p}{q} = \dfrac{2}{3}$, then $\dfrac{-p}{-q} = \dfrac{-2}{-3} = \dfrac{2}{3}$.

   A) $-\dfrac{2}{3}$ B) $\dfrac{-2}{3}$ C) $\dfrac{2}{-3}$ D) $\dfrac{2}{3}$

   6. D

7. The number of 5 kg weights and 10 kg weights I have is $4w$ and $2w$, respectively. Hence, $5(4w)+10(2w) = 200$, so $40w = 200$ and $w = 5$.

   A) 4 B) 5 C) 10 D) 20

   7. B

8. $(3x^3 - 4x^2) + (2x^2 - 3x) - (3x^3 - 4) = 3x^3 - 4x^2 + 2x^2 - 3x - 3x^3 + 4 = -2x^2 - 3x + 4$.

   A) $2x^2 - 3x - 4$ B) $2x^2 - 3x + 4$ C) $-2x^2 - 3x - 4$ D) $-2x^2 - 3x + 4$

   8. D

9. Since $3x + 10 = (3x - 4) + 14$, $3x + 10$ is odd. (Odd # + 14 = odd #.)

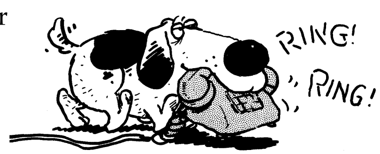
   A) positive B) prime C) odd D) even

   9. C

10. Yesterday the phone rang at 4 PM or later 80% of the time it rang, and it rang 50 times before 4 PM. Those 50 rings are 20% of all the rings. Thus, the phone rang 250 times yesterday.

    A) 200 B) 250 C) 300 D) 400

    10. B



11. Let the ages of the 5 trees be t, $t - 2$, $t - 4$, $t - 6$, $t - 8$. Then $t + (t - 2) + (t - 4) + (t - 6) + (t - 8) = 4440$. Thus, $5t - 20 = 4440$, and $t = 892$.

    A) 884 B) 888 C) 890 D) 892

    11. D

12. A line that passes through the points (p, q) and $(2p, 3q)$ has slope $(3q - q)/(2p - p) = 2q/p$. The slope between (p, q) and $(3p, 5q)$ is also $2q/p$.

    A) $(3p, 4q)$ B) $(3p, 5q)$ C) $(4p, 6q)$ D) $(4p, 8q)$

    12. B

13. The multiples of 3 between -9 and 12 include 0, so their product is 0.

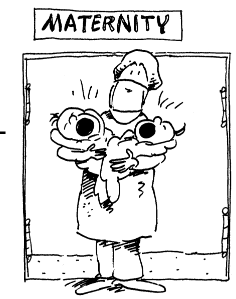
    A) -314 928 B) -2916 C) 0 D) 2916

    13. C

14. Of children born at the maternity ward yesterday, the ratio of boys to girls was $3x{:}4y = 5{:}6$. Thus, $18x = 20y$ or $9x = 10y$. Hence, $x{:}y = 10{:}9$.

    A) 10:9 B) 24:15 C) 15:24 D) 4:5

    14. A



15. $$\frac{\left(x^{200}\right)^{400}}{\left(x^{100}\right)^{200}} = \frac{x^{80\,000}}{x^{20\,000}} = x^{60\,000}.$$

    A) x^4 B) x^6 C) $x^{40\,000}$ D) $x^{60\,000}$

    15. D

16. If the average of x, y, and z is 16, their sum is $3(16) = 48$. If the average of x and y is 12, their sum is $2(12) = 24$. Hence $z = 48 - 24 = 24$.

    A) 4 B) 14 C) 20 D) 24

    16. D

17. Both $6n^8$ and $10n^{12}$ are factors of $30n^{12}$, the lcm.

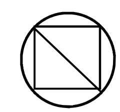
    A) $2n^8$ B) $30n^{12}$ C) $30n^{24}$ D) $60n^{96}$

    17. B

18. If the perim. is 64, each side has length 16. By Pythag. Th., a diameter is $16\sqrt{2}$. The area is $(8\sqrt{2})^2\pi = 128\pi$.

    A) 16π B) 32π C) 64π D) 128π

    18. D



19. Since $(x - y)^2 = 3^2$, $x^2 + y^2 - 2xy = 9$. Hence $485 - 2xy = 9$, and $xy = 238$.

    A) 162 B) 238 C) 482 D) 3880

    19. B

20. The roots of $(x - 1)(x + 2)(x - 3) \times \ldots \times (x - 19)(x + 20)(x - 21) = 0$ are 1, -2, 3, -4, . . . , 19, -20, and 21. Their sum is $(1 - 2) + (3 - 4) + \ldots + (19 - 20) + 21 = -10 + 21 = 11$.

    A) 10 B) 11 C) 21 D) 31

    20. B

21. $|4x| + 4|-x| = 4|x| + 4|x| = 8|x|$.

    A) 0 B) 8 C) $8|x|$ D) $4|4x|$

    21. C

22. $\sqrt{36^{64}} = \sqrt{\left(36^{32}\right)\left(36^{32}\right)} = 36^{32}$.

    A) 6^8 B) 6^{32} C) 36^8 D) 36^{32}

    22. D