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Contest # 5

Problem 5-1

Since making N's hundreds' digit a 9 and its units' digit a 0 will maximize the difference, no matter what the middle digit is, N-M = 9.20 - 0.29 = 891.

Problem 5-2

Since ab = 5, and *a* and *b* are integers, (*a*,*b*) must be one of (1,5), (5,1), (-1,-5), or (-5,-1). The least value of a^{b} is $(-1)^{-5} = \boxed{-1}$.

Problem 5-3

By trial and error, take each positive perfect square and subtract 1 to find every positive integer *n* for which n+1 is a perfect square. Find the least of those integers *n* for which 2n+1 is the square of an integer. Let's try 1. That fails because 1-1 = 0 is not positive. Try 4: 4-1 = 3, but 2(3)+1 = 7, which is not a perfect square. We try 9, then 16, and when we finally try 25, we get 25-1 = 24, and 2(24)+1 = 49. Both n+1 and 2n+1 are perfect squares when $n = \boxed{24}$.

Problem 5-4

If *B* is the number of Borgs, and *C* is the number of Corgs, then the number of Corgs is 2B+13. Therefore, B+(2B+13) = 100, B = 29, and $C = \boxed{71}$.

Problem 5-5

Whenever a > 1 and b > 1, $\log_a b = \frac{1}{\log_b a}$. We can write $\log_x 4 + \log_x 9 + \log_x 16 + \ldots + \log_x 2019^2 = k$, or $\log_x (2^2 \times 3^2 \times 4^2 \times \ldots \times 2019^2) = k$. It follows that $x^k = (2^2 \times 3^2 \times 4^2 \times \ldots \times 2019^2) = (2019!)^2$, so k = 2.

Problem 5-6

Method I: The square's side is $8\sqrt{3}$, so its diagonal is $8\sqrt{6} = d$. The trapezoid's height is $\frac{d}{4}$. Its longer base is $\frac{d}{2}$. The shaded \triangle 's legs trisect the diagonal (to prove this, use similar triangles I and II outlined in the middle diagram), so the shorter base is $\frac{d}{3}$. The trapezoid's area is $\frac{h}{2}(b_1+b_2) = \frac{d}{8}(\frac{d}{2}+\frac{d}{3}) = \frac{5d^2}{48} = 40$.

Method II: Two of the shaded triangle's vertices are midpoints of sides of the square, so the smaller un-

shaded region is 1/8 of the square, and the other unshaded regions are each 1/4 of the square. The shaded triangle's area is $(3/8) \times 192 = 72$. Notice that ΔI is similar to ΔII . Since M is a midpoint, the parallel sides of \triangle s I and II are in the ratio x:2x = 1:2, as are all corresponding sides. Thus, the smaller shaded triangle and the larger shaded triangle are similar, with ratio of similitude 2k:3k = 2:3. Since the area ratio of similar triangles is the square of their ratio of similitude, the ratio of the areas of the shaded triangles is $(2/3)^2 = 4:9$. The smaller shaded triangle's area = $(4/9) \times 72 = 32$.





The trapezoid's area is what remains: 72-32 = 40.

Contests written and compiled by Steven R. Conrad, Daniel Flegler, & Adam Raichel ©2019 by Mathematics Leagues Inc.